

Remarks on Mr. Denning's Paper. By Captain G. L. Tupman.

The radiant-point of a meteoric shower very seldom indicates the true direction of motion. When the orbital motion of the Earth is transverse to the true direction of motion of the stream, the radiant is displaced to the extent of 35° towards the apex of the Earth's way (a point in the ecliptic approximately 90° of longitude behind the Sun), on the supposition of the greatest velocity that can reasonably be assigned to meteors—that proper to parabolic motion—which for the present it will be sufficient to consider.

All the meteor streams we are certainly cognisant of at the present time can be accounted for by supposing them to belong to the solar system in the same manner as do comets. In the case of certain streams, as the *Leonids* of November 13, and the *Perseids* of August 10–11, the identity of their orbits with those of known comets is too certainly established to need comment, and these may be taken as typical streams belonging to the solar system.

The spreading out of the individuals in directions perpendicular to the general direction of motion necessarily involves a different orbit for each individual, and in proportion as the spreading out becomes more considerable, there will be less similarity of orbit.

The term *stream* or *shower* should properly be applied to the *ensemble* of a number of particles moving approximately in the same orbit in nearly the same periodic time. It is evident that we can have no knowledge whatever except of those streams that actually intersect the orbit of the Earth. There is a special case in which the Earth can remain in a stream for several weeks with a nearly fixed radiant. The orbit must nearly coincide with the plane of the ecliptic, the perihelion distance of the central portion be a little less than unity, and the motion be direct. The position of the radiant would be 90° before the Sun at the middle time.

If the perihelion distance be supposed to diminish, say to $\cdot 8$, the Earth would pass through the central part of the stream twice at an interval of about $3\frac{1}{2}$ months. At the first passage the radiant would be about 125° before the Sun, at the second about 30° , and therefore only to be observed shortly before sunrise. In all other cases the reappearance of a radiant after an interval indicates a distinct shower.

The results obtained by Mr. Denning after so much labour are probably to be attributed to the method employed in obtaining the radiants. The method was begun by Heis, and used by Greg, Herschel, Schiaparelli, and others, and was indeed the only method available with few observations. The observations of many nights were combined to obtain the apparent radiation, and the duration was supposed to begin and end with the first and last day so employed.

It is easy to obtain a radiant in any part of the sky, especially near the apex of the Earth's way, by collecting, night after night, meteors whose tracks, carried backwards, pass near the point in question.

I understand that Mr. Denning's coincidences in no degree depend upon previous knowledge; each is an entirely independent deduction, and therefore much more striking and singular.

We are now, probably, in possession of sufficient observations to enable the radiants proper to each day of the year to be deduced separately, and the number of determinations from multiple observations of solitary bright meteors is steadily increasing. These last are the best of all.

At present we are not obliged to resort to the idea of immense streams of meteors existing in space, and brought into collision with us by the motion of the Sun. The radiant-point of such a stream, moreover, would not be stationary many weeks.

Until the durations of fixed radiants be established in a more certain manner than those in the paper before us, we need not endeavour to explain them.

On a Small Term of Long Period in the Mean Motion of the Moon.

By E. Neison, Esq.

The mean motion of the Moon contains a term of long period, with an argument of the form

$$\{[a - 29n'' + 26n'] t + \epsilon - 29\epsilon'' + 26\epsilon' - E - A_0\},$$

where $(at + \epsilon - A_0)$ denotes the mean anomaly of the Moon, and ' and '' quantities relating to *Venus* and the Earth respectively. This term has a period of 130 years, and it appeared probable that its coefficient would be sensible, for it was evident that the terms depending on the eccentricities would not suffer the internal mutual destruction which occurs in the case of the analogous *Venus* term discovered by Hansen. The value of the term was calculated, but the relative positions of the perihelia of the Earth and *Venus* render its coefficient only one-fifth of that it would have possessed had it arisen from the planet *Mars*.

It is, however, very desirable that, when the value of such a term has been determined, it should be placed on record in sufficient detail to save the useless labour of a repetition of the work by subsequent investigators in the same field. Moreover, small as the term is, it might well be included in any future lunar tables, for it has a comparatively short period.

Neglecting terms depending on the Moon's longitude and latitude, then the disturbing function may be written (Delaunay, *Conn. des Temps*, Add. p. 8, 1862)

$$R^0 = m' r^2 \cdot \left\{ \frac{1}{4\Delta^3} - \frac{3}{2} \cdot \frac{1}{\Delta^5} \cdot \eta_1^2 \sin(v' - B') \cdot r' [r'' \sin(v'' - B') + 2r' \sin(v' - B')] \right\},$$

the notation being slightly altered to bring it into accordance with my other result. Suppose the terms of the third order of the eccentricities and inclinations to be neglected, except as contained in the function Δ , and replace Δ^{-s} by its general term,

$$\Delta^{-s} = (a')^{-s} \cdot [B_{\frac{s}{2}}^s]_{i+k}^i \cdot \cos \{[in'' - (i+k)n']t + E\};$$

then the disturbing function becomes

$$\begin{aligned} R = m' (a')^{-3} r^2 & \left\{ \frac{1}{4} [B_{\frac{3}{2}}]^i_{i+k} \cos \{ [in'' - (i+k) n'] t + E \} \right. \\ & \left. + \frac{3}{8} \eta_1^2 \left(\alpha^2 [B_{\frac{5}{2}}]^i_{i+k \pm 2} + 2\alpha [B_{\frac{5}{2}}]^{i+1}_{i+k-1} + 2\alpha [B_{\frac{5}{2}}]^{i-1}_{i+k+1} \right) \cos \{ [in'' - (i+k) n'] t + E \pm 2B_1 \} \right\} \end{aligned}$$

Giving i and k the values necessary to bring the indices of $[B_{\frac{5}{2}}]$ to the values 26 and 29, then

$$\begin{aligned} R = m' (a')^{-3} r^2 & \left\{ \frac{1}{4} [B_{\frac{3}{2}}]_{26}^{29} \cos \{ [29n'' - 26n'] t + E \} \right. \\ & \left. + \frac{3}{8} \eta_1^2 \left(\alpha^2 [B_{\frac{5}{2}}]_{28}^{29} + 2\alpha [B_{\frac{5}{2}}]_{27}^{28} \right) \cos \{ [29n'' - 26n'] t + E - 2B_1 \} \right\} \end{aligned}$$

Terms of the fourth order have been neglected, and a represents, as usual, the ratio $\frac{a''}{a'}$.

If (R) denotes the term in brackets, R may be written

$$R = m' (a')^{-3} r^2. (R). \cos \{ [29n'' - 26n'] t + E_1 \}.$$

Replacing r^2 by its value, and writing $m' \frac{a^3}{a'^3} = \sigma m^2$, and

$$\delta n = -3 \iint a n^2 \left(\frac{dR}{d\epsilon} \right) d\tau^2 = \sigma m^2 \cdot \left(\frac{n}{p} \right)^2 \cdot 3e \cdot (R) \sin \{ [\alpha - 29n'' + 26n'] t + \epsilon - E_1 \}.$$

The coefficient of t in the argument is $p = 10,171''$. Therefore the value of $\sigma m^2 \left(\frac{n^2}{p^2} \right) 3e$ is $1,378''$. The value of $[B_{\frac{1}{2}}]$ can be found in the usual manner, from either Delaunay or Pontecoulant. Substituting the proper values in the expression for δn , it becomes

$$\begin{aligned} \delta n = & +0''.005 \sin \{ [\alpha - 29n'' + 26n'] t + \epsilon - 29\epsilon'' + 26\epsilon' - A + 3\varpi' \} \\ & - 0''.067 \sin \left\{ \begin{array}{ccc} & , & \\ & , & + 2\varpi' - \varpi'' \end{array} \right\} \\ & + 0''.201 \sin \left\{ \begin{array}{ccc} & , & \\ & , & + \varpi' - 2\varpi'' \end{array} \right\} \\ & - 0''.189 \sin \left\{ \begin{array}{ccc} & , & \\ & , & + 3\varpi'' \end{array} \right\} \\ & + 0''.049 \sin \left\{ \begin{array}{ccc} & , & \\ & , & + \varpi' + 2B' \end{array} \right\} \\ & - 0''.122 \sin \left\{ \begin{array}{ccc} & , & \\ & , & + \varpi'' + 2B' \end{array} \right\} \\ = & - 0''.100 \sin \{ [\alpha - 29n'' + 26n'] t + \epsilon - 29\epsilon'' + 26\epsilon' - A_0 + 18^{\circ}.5 \}. \end{aligned}$$

By taking into consideration higher powers of the inclination, and the ordinary perturbations of the Moon, the coefficient of the term will be raised to nearly $-0''.150$.

On a Secular Term in the Mean Motion of the Moon.

By E. Neison, Esq.

The lunar co-ordinates contain a term of long period due to the action of *Mars*, and with the argument

$$\{[A - 15n'' + 28n'''] t + E\},$$

where At denotes the mean motion of the Moon's perigee, and $n''t$ and $n'''t$ the mean motions of the Earth and *Mars*. This term has the very long period of 6,000 years, and is of the 14th order of the eccentricity of *Mars* and the Earth. The eccentricity of *Mars* is, however, so large that the coefficient of this term is comparatively considerable, despite its high order.

This term will introduce a similar term into the mean motion of the Moon, through the means of the solar perturbations; and this term will be multiplied by the square of the lunar perturbation arising from the action of the Sun, but divided by the cube of the small coefficient of the time. As

$$[A - 15n'' + 28n'''] = +197''.715,$$

the term will rise immensely, and will have, apparently, a considerable coefficient. Its calculation will be long and laborious, but, unless its terms destroy each other in some unexpected manner, it will repay the labour.

The present note, however, is directed to a single term with this general argument. The term

$$[A - 15n'' + 28n'''] - 9\varpi''' - 5\varpi''$$

has the coefficient of t reduced to zero from the motion of the perihelia of *Mars* and the Earth. It forms, therefore, a secular term in the mean motion of the Moon, and is, I believe, the first term of this kind due to the direct action of the planets which has been made known. It will obviously increase the secular acceleration by a small fraction, for it will appear in the mean motion as multiplying the square of the time.

Professor Newcombe is, I believe, the first who discovered that the mean motion of the Moon contains terms depending on the motion of the Moon's perigee, a result which I independently found two years back.

I know of at least two similar secular terms in the Moon's mean motion, but they will be much smaller, although they contain the Moon's mean motion in their arguments, for they depend on terms involving products of the fifth order of the lunar eccentricity and inclination.

The existence of these curious terms seemed of sufficient interest to merit their existence being announced.